

**WMAP and supergravity inflationary models**M.C. Bento,\* N. M. C. Santos,<sup>†</sup> and A. A. Sen<sup>‡</sup>*Departamento de Física, Instituto Superior Técnico, Av. Rovisco Pais 1, 1049-001 Lisboa, Portugal*

(Received 7 July 2003; published 29 January 2004)

We study a class of  $N=1$  supergravity inflationary models in which the evolution of the inflaton dynamics is controlled by a single power in the inflaton field at the point where the observed density fluctuations are produced, in the context of the braneworld scenario, in light of Wilkinson Microwave Anisotropy Probe results. In particular, we find that the bounds on the spectral index and its running constrain the parameter space both for models where the inflationary potential is dominated by a quadratic term and by a cubic term in the inflaton field. We also find that  $\alpha_s > 0$  is required for the quadratic model whereas  $\alpha_s < 0$  for the cubic model. Moreover, we have determined an upper bound on the five-dimensional Planck scale,  $M_5 \lesssim 0.019 M$ , for the quadratic model. On the other hand, a running spectral index with  $n_s > 1$  on large scales and  $n_s < 1$  on small scales is not possible in either case.

DOI: 10.1103/PhysRevD.69.023508

PACS number(s): 98.80.Cq

**I. INTRODUCTION**

The first year Wilkinson Microwave Anisotropy Probe (WMAP) data have confirmed the “concordance” values of the cosmological parameters with unprecedented accuracy and given important information on the primordial spectrum of density perturbations [1,2]. Their results favor Gaussian, purely adiabatic fluctuations and a spectral index that runs from  $n_s > 1$  on large scales to  $n_s < 1$  on small scales. Moreover, WMAP has confirmed earlier Cosmic Background Explorer (COBE) Differential Microwave Radiometer (DMR) observations that there is a lower amount of power on the largest scales when compared to that predicted by the standard cold dark matter model with a cosmological constant ( $\Lambda$ CDM).

Although these results are not yet firmly established (for an analysis of WMAP results which finds no evidence of running see Ref. [3]), it seems worthwhile to reexamine inflationary models in light of WMAP results, as they may give us further insight into the very early Universe. Supergravity inflationary models are particularly important as supersymmetry (or its local version, supergravity) is the only known way to avoid the hierarchy problem, i.e. the fact that the high energy scale of inflation communicates to other sectors of the theory driving the electroweak scale much above its observed value via radiative corrections.

However, supergravity inflationary models also suffer from a kind of hierarchy problem as supersymmetry is broken by the large cosmological constant during inflation giving all scalars, including the inflaton, a soft mass of the order of the Hubble parameter [4]. As a result, the curvature of the inflaton potential, as measured by the  $\eta$  slow-roll parameter, becomes too large to allow for a sufficiently long period of

inflation to take place—the so-called  $\eta$  problem.

Recently, it has been shown that this problem can be avoided within the Randall-Sundrum type II braneworld scenario [5], at least for a class of supergravity models in which the evolution of the inflaton dynamics is controlled by a single power at the point where the observed density fluctuations are produced and the inflationary potential can, therefore, be approximately given by

$$V \simeq \Delta^4 \left[ 1 + c_n \left( \frac{\phi}{M} \right)^n \right], \quad (1)$$

where  $M = M_P / \sqrt{8\pi}$  is the reduced Planck mass. In the braneworld context, the Friedmann equation acquires an additional term quadratic in the energy density [6]

$$H^2 = \frac{8\pi}{3M_P^2} \rho \left[ 1 + \frac{\rho}{2\lambda} \right], \quad (2)$$

where  $\lambda$  is the brane tension, which relates the four and five-dimensional Planck scales through

$$M_P = \sqrt{\frac{3}{4\pi}} \frac{M_5^3}{\sqrt{\lambda}}. \quad (3)$$

It is precisely the new parameter  $M_5$  that plays a crucial role in the resolution of the  $\eta$ -problem in supergravity inflation. As shown in Ref. [7], for the case where the first term in Eq. (1) is dominant and  $n=2$  or  $n=3$ , this problem can be avoided provided the five-dimensional Planck mass satisfies, respectively, the condition  $M_5 \lesssim 10^{16}$  GeV and  $M_5 \approx 1.1 \times 10^{16}$  GeV. The case where the second term is dominant and  $n=2$ , corresponding to chaotic inflation, has been studied in Refs. [8,9], where it is shown that it is possible to achieve successful inflation with sub-Planckian field values, thereby avoiding well known difficulties with higher order nonrenormalizable terms.

In this paper, we reexamine this class of supergravity models for the quadratic and cubic cases in light of WMAP results. The case of chaotic inflation has already been analyzed in Ref. [10], with the conclusion that the quadratic

\*Also at CFIF, Instituto Superior Técnico, Lisboa. Email address: bento@sius.ist.utl.pt

<sup>†</sup>Also at CFIF, Instituto Superior Técnico, Lisboa. Email address: ncsantos@gtae3.ist.utl.pt

<sup>‡</sup>Also at CENTRA, Instituto Superior Técnico, Lisboa. Email address: anjan@x9.ist.utl.pt

potential is allowed at two-sigma for any value of the brane tension and the quartic potential is very constrained, particularly in the case where the inflationary energy scale is close to the brane tension. Here we concentrate on the case where the first term in Eq. (1) is dominant. We find that although the running of the scalar spectral index is within the bounds determined by WMAP for both the quadratic and cubic models and for a wide range of potential parameters, a spectral index running from  $n_s > 1$  on large scales to  $n_s < 1$  on small scales is not possible in either case as precisely the opposite trend is found.

## II. QUADRATIC POTENTIAL

We first consider the case where the potential is quadratic in the inflaton field,  $\phi$ , and we rewrite it as

$$V = V_0 + \frac{1}{2} m^2 \phi^2, \quad (4)$$

and assume that the first term is dominant.

In supergravity, effective mass squared contributions of fields are given by

$$\frac{1}{2} m^2 = 8\pi \frac{V_0}{M_P^2} \approx 3H^2, \quad (5)$$

since the horizon of the inflationary de Sitter phase has a Hawking temperature given by  $T_H = H/2\pi$  [4].

Contributions like the ones of Eq. (5) lead to  $\eta \equiv M_P^2 V''/8\pi V \approx 2$ ; however, the onset of inflation requires  $\eta \ll 1$ . Within the braneworld scenario, however,  $\eta$  and the remaining “slow-roll” parameters  $\epsilon$  and  $\xi$ , are modified, at high energies, by a factor proportional to  $\lambda/V$  [8]

$$\epsilon \equiv \frac{M_P^2}{16\pi} \left( \frac{V'}{V} \right)^2 \frac{1 + V/\lambda}{(1 + V/2\lambda)^2}, \quad (6)$$

$$\eta \equiv \frac{M_P^2}{8\pi} \left( \frac{V''}{V} \right) \frac{1}{1 + V/2\lambda}, \quad (7)$$

$$\xi \equiv \frac{M_P^4}{(8\pi)^2} \frac{V' V'''}{V^2} \frac{1}{(1 + V/2\lambda)^2}. \quad (8)$$

In the high energy approximation,  $V \gg \lambda$ , we obtain, for this model

$$\epsilon = \frac{M_P^2}{8\pi} \frac{m^4 \phi^2}{V_0^2 \alpha}, \quad (9)$$

$$\eta = \frac{4}{\alpha}, \quad (10)$$

$$\xi = 0, \quad (11)$$

where we have also used the approximation  $V \approx V_0$  during inflation and the definition  $\alpha \equiv V_0/\lambda$ . As shown in Ref. [7],

if  $\alpha$  is sufficiently large, the  $\eta$ -problem is automatically solved by the brane correction.

The number of  $e$ -folds during inflation,  $N$ , in the brane-world scenario, is given by [8]

$$N \approx -\frac{8\pi}{M_P^2} \int_{\phi_I}^{\phi_F} \frac{V}{V'} \left[ 1 + \frac{V}{2\lambda} \right] d\phi, \quad (12)$$

in the slow-roll approximation. We see that, as a result of the modification in the Friedmann equation, the expansion rate is increased, at high energies, by a factor  $V/2\lambda$ . For this model, we get, in the high-energy approximation,

$$N = \frac{\alpha}{4} \log \left( \frac{\phi_I}{\phi_F} \right) + \frac{2\pi\alpha}{M_P^2} (\phi_I^2 - \phi_F^2) + \frac{16\pi^2\alpha}{M_P^4} (\phi_I^4 - \phi_F^4). \quad (13)$$

Notice that, for sub-Planckian field values, the second and third terms are negligible. The value of  $\phi$  at the end of inflation can be obtained from the condition

$$\max\{\epsilon(\phi_F), |\eta(\phi_F)|\} = 1. \quad (14)$$

However, we shall consider  $\phi_F = \beta M$  as a free parameter since quadratic potentials of the type we are studying arise typically in the context of hybrid inflation, once some other field is held at the origin by its interaction with  $\phi$ . In these scenarios, inflation may end due to instabilities triggered by the dynamics of the other field and, therefore, the amount of inflation strongly depends on the value of the inflaton field at the end of inflation,  $\phi_F$ , at the time the instabilities arise. Actually, these instabilities are necessary in order to end inflation as  $\epsilon \ll 1$  for  $\alpha \gg 1$  and sub-Planckian field values.

The amplitude of scalar perturbations is given by [8]

$$A_s^2 \approx \left( \frac{512\pi}{75M_P^6} \right) \frac{V^3}{V'^2} \left[ 1 + \frac{V}{2\lambda} \right]^3 \Big|_{k=aH}, \quad (15)$$

where the right-hand side should be evaluated as the comoving wave number equals the Hubble radius during inflation,  $k = aH$ . Thus the amplitude of scalar perturbations is increased relative to the standard result at a fixed value of  $\phi$  for a given potential. Using the high energy approximation and  $V \approx V_0$  in Eq. (15), we obtain

$$A_s^2 \approx \frac{1600\pi}{75} \frac{V_0^6}{\lambda^3 m^4} \exp \left( \frac{-8N_k}{\alpha} \right), \quad (16)$$

where  $N_k$  is the number of  $e$ -folds between the time the scales of interest leave the horizon and the end of inflation.

The scale-dependence of the perturbations is described by the spectral tilt [8]

$$n_s - 1 \equiv \frac{d \ln A_s^2}{d \ln k} \approx -6\epsilon + 2\eta, \quad (17)$$

which, for this model, gives

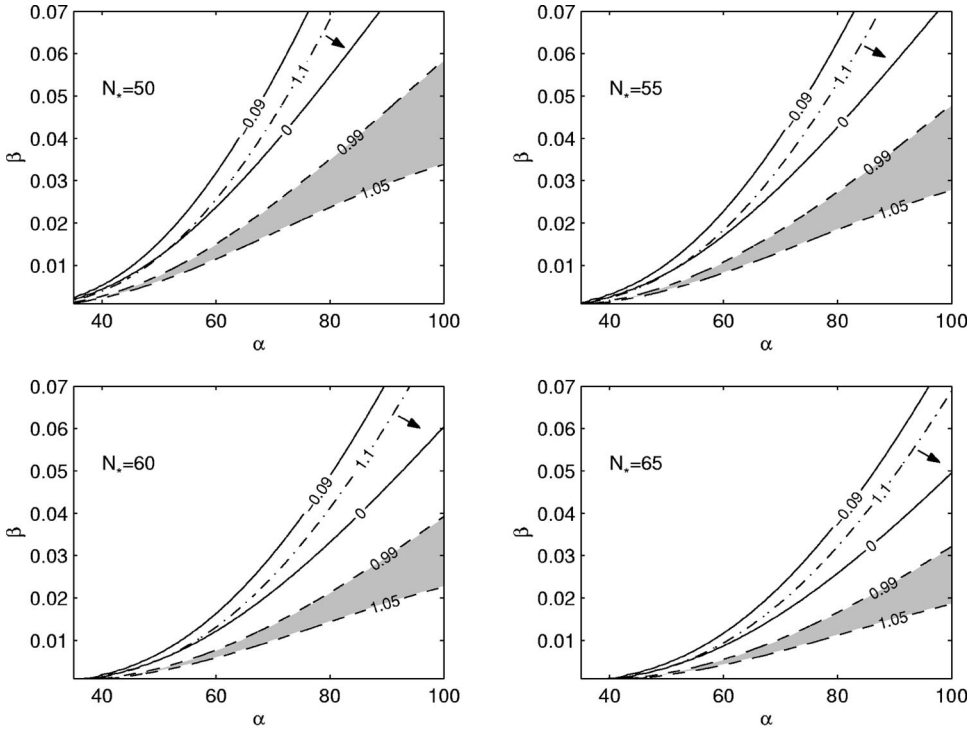


FIG. 1. Contours of the inflationary observables  $n_s$  (dashed),  $\alpha_s$  (full) and  $r_s$  (dot-dashed) in the  $(\alpha, \beta)$  plane for the quadratic model; the arrow points in the direction of decreasing  $r_s$ . The allowed region is shaded and, as indicated, different panels correspond to different values of  $N_*$ .

$$n_s = 1 - 192 \frac{\beta^2}{\alpha^2} \exp\left(\frac{8N_k}{\alpha}\right) + \frac{8}{\alpha}. \quad (18)$$

The “running” of the scalar spectral index is given by

$$\alpha_s \equiv \frac{dn_s}{d \ln k} = 16\epsilon\eta - 24\epsilon^2 - 2\xi, \quad (19)$$

and we get

$$\alpha_s = 512 \frac{\beta^2}{\alpha^2} \exp\left(\frac{8N_k}{\alpha}\right) \left[1 - 3 \exp\left(\frac{8N_k}{\alpha}\right)\right]. \quad (20)$$

The amplitude of tensor perturbations is given by [11]

$$A_t^2 = \frac{64}{150\pi M_p^4} V \left(1 + \frac{V}{2\lambda}\right) F^2 \Big|_{k=aH}, \quad (21)$$

where

$$F^2 = \left[ \sqrt{1+s^2} - s^2 \sinh^{-1}\left(\frac{1}{s}\right) \right]^{-1}, \quad (22)$$

and

$$s \equiv \left[ \frac{2V}{\lambda} \left(1 + \frac{V}{2\lambda}\right) \right]^{1/2}. \quad (23)$$

In the low energy limit ( $s \ll 1$ ),  $F^2 \approx 1$ , whereas  $F^2 \approx 3V/2\lambda$  in the high energy limit. Defining (we choose the normalization of Ref. [12])

$$r_s \equiv 16 \frac{A_t^2}{A_s^2}, \quad (24)$$

we obtain

$$r_s \approx 0.06 \frac{M_p^4 m^4 \lambda \beta^2}{V_0^3} \exp\left(\frac{8N_k}{\alpha}\right). \quad (25)$$

WMAP bounds on the above inflationary observables are, for this class of models (case  $\eta > 3\epsilon$ , class D in Ref. [12])

$$0.99 < n_s < 1.28, \quad -0.09 \leq \alpha_s \leq 0.03,$$

$$r_s \leq 1.10. \quad (26)$$

These bounds refer to the scale best probed by the CMB observations i.e.  $k = 0.002 \text{ Mpc}^{-1}$ ; accordingly, we set  $N_k(k=0.002) = N_*$ . On the other hand, bounds on  $n_s$  from other experiments are less blue, e.g. the combined data sets from BOOMERANG, CBI, DASI, DMR, MAXIMA, TOCO and VSA give [13]

$$0.955 < n_s < 1.05. \quad (27)$$

In Fig. 1, we show contours of the observational bounds on the inflationary observables  $n_s$ ,  $r_s$  and  $\alpha_s$  in the  $(\alpha, \beta)$  parameter space, for different values of  $N_*$ ; in these plots, we have taken the bounds of Eq. (26) except for the upper bound on  $n_s$ , for which we took the bound of Eq. (27) instead since such a blue spectrum is not to be expected. We also plotted the  $\alpha_s = 0$  contour, which shows that  $\alpha_s$  is required to be positive for this model. The shaded area corresponds to the allowed region in parameter space. We would also like to mention that we have checked that it is possible to obtain sufficient inflation with sub-Planckian field values e.g.  $N = 70$  for  $\phi_I = 0.2M_p$  and  $\alpha, \beta$  within the range specified in Fig. 1.

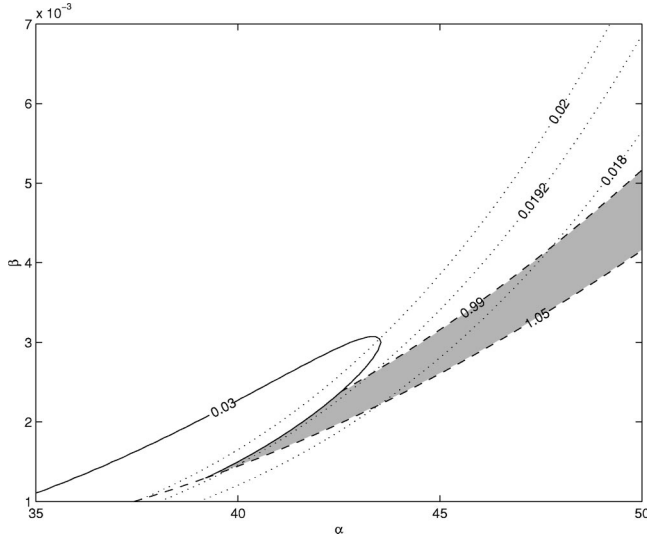


FIG. 2. Contours of the inflationary observables  $n_s$  (dashed) and  $\alpha_s$  (full) for  $N_*=55$ . The allowed region is shaded and the dotted contours correspond to different values of  $M_5$  in units of  $M$ .

Notice that the contour corresponding to the upper bound on  $\alpha_s$  is too small to be visible in Fig. 1, hence we show it in Fig. 2, for  $N_*=55$  (similar behavior is obtained for other values of  $N_*$ ), which makes it clear that this bound plays an important role in constraining the parameter space. We obtain lower bounds on  $\alpha$  and  $\beta$ , namely  $\alpha > 39.23$ , practically independent of  $N_*$ , whereas the lower bound on  $\beta$  ranges from  $2.2 \times 10^{-3}$  to  $1.7 \times 10^{-4}$  as  $N_*$  ranges from 50 to 75.

In Figs. 2 and 3, we have superposed contours of the scale  $M_5$ , as derived from Eqs. (16) and (3), where we have used the COBE normalization i.e.  $A_s = 2 \times 10^{-5}$  for  $N_k = N_*$ . As the allowed region is quite narrow for low values of  $\alpha$ , it allows us to find an upper bound on  $M_5$ ,  $M_5 \lesssim 0.0194 M$ , which we have checked is almost independent of  $N_*$ . Combining the above results, we find a lower bound

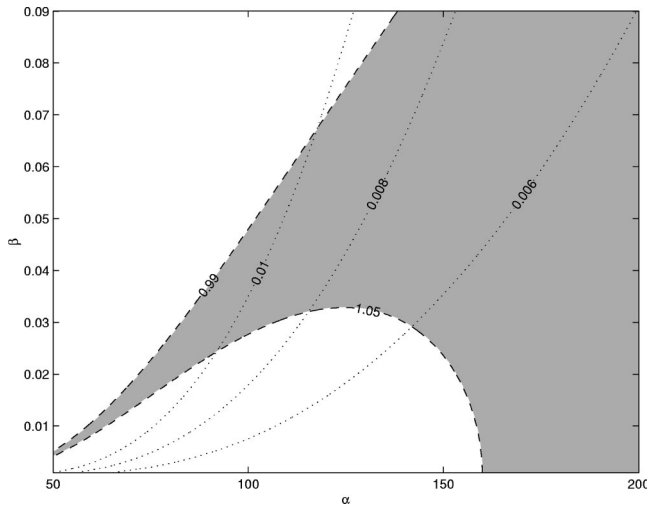


FIG. 3. Contours of the inflationary observable  $n_s$  (dashed) for  $N_*=55$  (notice that the range of  $\alpha$  and  $\beta$  is enlarged as compared with Fig. 2). The allowed region is shaded and the dotted contours correspond to different values of  $M_5$  in units of  $M$ .

on the scale  $V_0$ , namely  $V_0^{1/4} \lesssim 2.1 \times 10^{-3} M$ .

Notice that we have chosen to vary  $N_*$  since, although a wide variety of assumptions about  $N_*$  can be found in the literature, the determination of this quantity requires a model of the entire history of the Universe. However, while from nucleosynthesis onwards this is now well established, at earlier epochs there are considerable uncertainties such as the mechanism ending inflation and details of the reheating process. This issue was recently reviewed in Ref. [14] (see also Ref. [15] for similar results), where a model-independent upper bound was derived, namely  $N_* < 60$ ; in fact,  $N_*=55$  is found to be a reasonable fiducial value with an uncertainty of around 5 around that value; however, the authors stress that there are several ways in which  $N_*$  could lie outside that range, in either direction. Moreover, in the braneworld context, one expects  $N_*$  to depend on the brane tension. Actually, one expects to obtain larger values of  $N_*$  because, in the high-energy regime, the expansion laws corresponding to matter and radiation domination are slower than in the standard cosmology, which implies a greater change in  $aH$  relative to the change in  $a$ , therefore requiring a larger value of  $N_*$ . This is confirmed by the results of Ref. [16], where the bound  $N_* < 75$  is found for brane inspired cosmology.

We have studied the dependence of  $n_s$  on  $k$  to see whether  $n_s$  can vary from  $n_s > 1$  to  $n_s < 1$  from large to small scales. From the condition  $k = aH$ , assuming that  $H$  is approximately constant during inflation, we obtain the relation

$$\exp(N_k) = \frac{k_F}{k}, \quad (28)$$

where  $k_F$  is the value of  $k$  at the end of inflation. Inserting this relation in Eq. (18), we obtain

$$n_s \approx 1 + \frac{8}{\alpha} - 192 \frac{\beta^2}{\alpha^2} \left( \frac{k_F}{k} \right)^{8/\alpha}, \quad (29)$$

from which we conclude that, for  $\alpha, \beta$  fixed,  $n_s$  increases with  $k$ . Hence, it is not possible to obtain the desired behavior, i.e.  $n_s$  decreasing from  $n_s > 1$  to  $n_s < 1$  as  $k$  increases.

### III. CUBIC POTENTIAL

We shall now consider the case where, due to some cancellation mechanism [17], the quadratic term is absent and the potential is cubic in  $\phi$ ,

$$V = \Delta^4 \left[ 1 + \gamma \left( \frac{\phi}{M} \right)^3 \right]. \quad (30)$$

As mentioned before, we shall assume that the first term is dominant. The parameter  $\gamma$  is expected to be of order unity and negative [17]; the model of Ref. [18] corresponds to precisely this case, with  $\gamma = -4$ .

We start by computing the slow-roll parameters,

$$\epsilon \approx \frac{18\gamma^2}{\alpha} \left( \frac{\phi}{M} \right)^4,$$

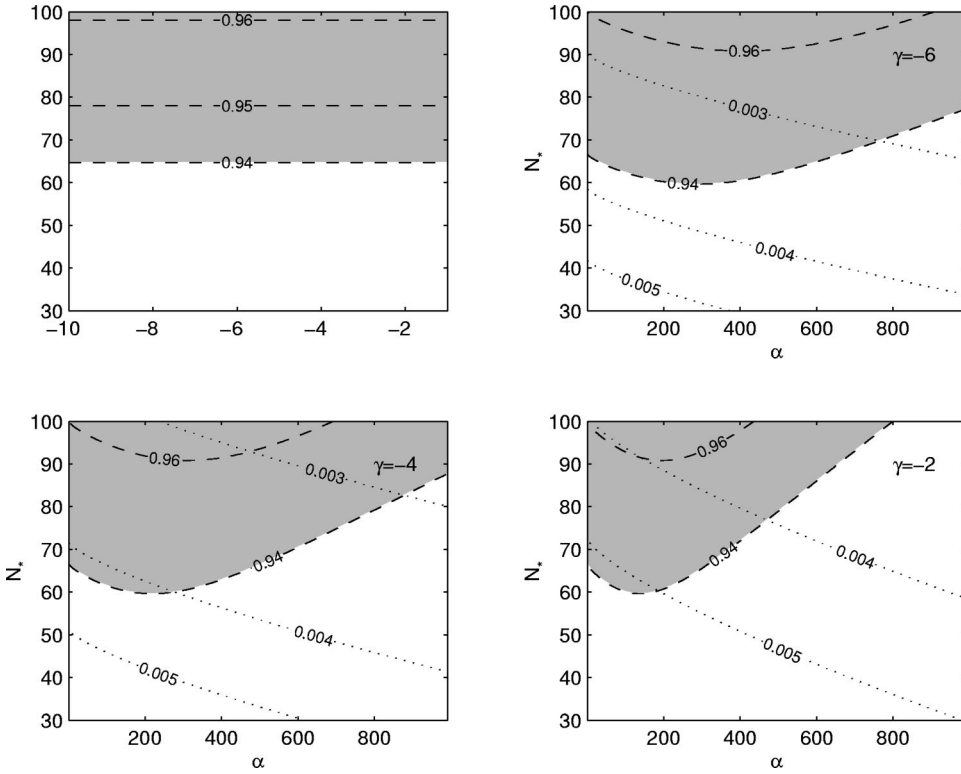


FIG. 4. Contours of the inflationary observables  $n_s$  (dashed) in the  $(\alpha, N_*)$  plane for the cubic model. The allowed region is shaded and we also show contours corresponding to different values of  $M_5$  (dotted) in units of  $M$ . The upper left panel corresponds to the results for this model in the low energy approximation, i.e. without the brane corrections, and the remaining three panels correspond to the high energy approximation for different values of  $\gamma$ .

$$\eta \simeq \frac{12\gamma}{\alpha} \left( \frac{\phi}{M} \right),$$

$$\xi \simeq \frac{72\gamma}{\alpha^2} \phi^2, \quad (31)$$

where  $\alpha \equiv \Delta^4/\lambda$ .

The value of  $\phi$  at the end of inflation can be obtained from Eq. (14); we get, from  $|\epsilon| \simeq 1$

$$\phi_F \simeq \left( \frac{\alpha}{18\gamma^2} \right)^{1/4} M, \quad (32)$$

while, from  $|\eta| \simeq 1$ , we obtain

$$\phi_F \simeq \left( \frac{\alpha}{12|\gamma|} \right) M. \quad (33)$$

Hence, the prescription to be used depends on the value of  $\alpha$ . For  $\gamma = -4$ , we see that the two prescriptions coincide for  $\alpha \simeq 26$ .

The number of  $e$ -foldings,  $N$ , is given by

$$N = \frac{\alpha M}{6|\gamma|} \left[ \frac{1}{\phi_I} - \frac{1}{\phi_F} \right]. \quad (34)$$

Therefore, sufficient inflation to solve the cosmological horizon/flatness problems, that is  $N > 70$ , is achieved, for instance for  $\gamma = -4$ , if  $\phi_I < 7.5 \times 10^{-2} M_P$ .

For  $A_s$ , we obtain, in the high energy regime,

$$A_s^2 \simeq \frac{\alpha^4 \lambda}{5400 \pi^2 \gamma^2 \phi_k^4}, \quad (35)$$

where  $\phi_k$  is the value of  $\phi$  at horizon-crossing. The scalar spectral index and its running can be readily computed from the slow-roll parameters, Eq. (31), via Eqs. (17) and (19). Notice that the inflationary observables can, of course, be written as a function of  $N_*$ , as for the quadratic model, using Eq. (34) with  $\phi_I = \phi_k$ , but one has to bear in mind that the prescription to use for  $\phi_F$  depends on  $\alpha$ .

WMAP bounds on the inflationary observables are, for this class of models (case  $\eta < 0$ , class A in Ref. [12])

$$0.94 < n_s < 1.00, \quad -0.02 \leq \alpha_s \leq 0.02,$$

$$r_s \leq 0.14, \quad (36)$$

again for the scale  $k = 0.002 \text{ Mpc}^{-1}$ . In Fig. 4, we show contours of the inflationary observable  $n_s$ ,  $\alpha_s$  in the  $(\alpha, N_*)$  plane. We have checked that neither  $\alpha_s$  nor  $r_s$  give further constraints on the parameter space. We also show contours corresponding to different values of  $M_5$ , as given by Eq. (35), again COBE normalized. The upper left panel corresponds to the results for the low energy regime,  $V \ll \lambda$ , where the brane corrections are negligible, and the remaining three panels correspond to the high energy regime,  $V \gg \lambda$ , for different values of  $\gamma$ .

We see that it is the lower bound on  $n_s$  that most constrains the model and, clearly,  $n_s = 1$  cannot be obtained. It is also clear that, for the model to work,  $N_* > 65$  is required if brane corrections are not included and  $N_* > 60$  if those corrections are included; in the latter case, however, this bound increases outside the range  $300 \lesssim \alpha \lesssim 100$ , for  $\gamma = -4$  (this



range is slightly  $\gamma$ -dependent, see Fig. 4). Moreover, the running parameter is always negative although it can be quite small. Finally,  $0.042 M \leq M_5 \leq 0.025 M$ , for  $\gamma = -4$  (however, these bounds do not change significantly with  $\gamma$ , see Fig. 4).

Clearly, the spectral index cannot run from  $n_s > 1$  on large scales to  $n_s < 1$  on small scales, since  $n_s(k) < 1$  for this model.

In Ref. [7], a very strict bound on  $\alpha$  was derived for this model from the requirement that the reheating temperature is small enough to avoid the gravitino problem. We should like to point out that there was a numerical error in that computation and, in fact, the bound is much weaker and practically meaningless.

#### IV. CONCLUSIONS

We have analyzed the implications of WMAP results, in particular the bounds on the inflationary observables, for a class of supergravity inflationary models, Eq. (1) with  $n = 2, 3$ . We find that, for the quadratic potential, the main constraints come from the WMAP's bounds on  $n_s$  and upper bound on  $\alpha_s$ . We have obtained lower bounds on parameters  $\alpha$  and  $\beta$ , namely  $\alpha \gtrsim 39$  (pratically independent of  $N_*$ ) and the lower bound on  $\beta$  ranges from  $2.2 \times 10^{-3}$  to  $1.7 \times 10^{-4}$

as  $N_*$  varies between 50 and 75. We have also found an upper bound on  $M_5$ ,  $M_5 \leq 0.0194 M$ , pratically independent of  $N_*$ . Moreover, we conclude that  $\alpha_s > 0$  is required for this model.

For the cubic potential, in the low energy regime i.e. without the brane correction, a relatively high value of  $N_*$ ,  $N_* > 65$ , is required so as to meet WMAP's lower bound on  $n_s$ . In the high energy regime, when brane corrections are significant, the allowed region in the  $(\alpha, N_*)$  parameter space changes with  $\gamma$  and the main constraints come from WMAP's lower bound on  $n_s$ . Moreover, we find that  $\alpha_s < 0$  for this model.

We have also studied whether it is possible to obtain a running spectral index such that  $n_s > 1$  on large scales and  $n_s < 1$  on small scales and concluded that this is not possible for either model.

#### ACKNOWLEDGMENTS

M.C.B. acknowledges the partial support of Fundação para a Ciência e a Tecnologia (Portugal) under the grant POCTI/1999/FIS/36285. The work of A.A.S. was fully financed by the same grant. N.M.C. Santos was supported by FCT grant SFRH/BD/4797/2001.

- 
- [1] C.L. Bennett *et al.*, *Astrophys. J., Suppl. Ser.* **148**, 1 (2003).
  - [2] D.N. Spergel *et al.*, *Astrophys. J., Suppl. Ser.* **148**, 175 (2003).
  - [3] V. Barger, H. Lee, and D. Marfatia, *Phys. Lett. B* **565**, 33 (2003); U. Seljak, P. McDonald, and A. Makarov, *Mon. Not. R. Astron. Soc.* **342**, L79 (2003).
  - [4] O. Bertolami and G.G. Ross, *Phys. Lett. B* **183**, 163 (1987).
  - [5] L. Randall and R. Sundrum, *Phys. Rev. Lett.* **83**, 4690 (1999).
  - [6] P. Binétruy, C. Deffayet, U. Ellwanger, and D. Langlois, *Phys. Lett. B* **477**, 285 (2000); T. Shiromizu, K. Maeda, and M. Sasaki, *Phys. Rev. D* **62**, 024012 (2000); E.E. Flanagan, S.H. Tye, and I. Wasserman, *ibid.* **62**, 044039 (2000).
  - [7] M.C. Bento, O. Bertolami, and A.A. Sen, *Phys. Rev. D* **67**, 023504 (2003).
  - [8] R. Maartens, D. Wands, B.A. Bassett, and I.P.C. Heard, *Phys. Rev. D* **62**, 041301 (2000).
  - [9] M.C. Bento and O. Bertolami, *Phys. Rev. D* **65**, 063513 (2002).
  - [10] A.R. Liddle and A.J. Smith, *Phys. Rev. D* **68**, 061301 (2003).
  - [11] D. Langlois, R. Maartens, and D. Wands, *Phys. Lett. B* **489**, 259 (2000).
  - [12] H.V. Peiris *et al.*, *Astrophys. J., Suppl. Ser.* **148**, 213 (2003).
  - [13] J.L. Sievers *et al.*, *Astrophys. J.* **591**, 599 (2003).
  - [14] A.R. Liddle and S.M. Leach, *Phys. Rev. D* **68**, 103503 (2003).
  - [15] S. Dodelson and L. Hui, *Phys. Rev. Lett.* **91**, 131301 (2003).
  - [16] B. Wang and E. Abdalla, hep-th/0308145.
  - [17] J.A. Adams, G.G. Ross, and S. Sarkar, *Phys. Lett. B* **391**, 271 (1997).
  - [18] G.G. Ross and S. Sarkar, *Nucl. Phys.* **B461**, 597 (1995).